NASA TECHNICAL TRANSLATION NASA TT F-15,072

DETERMINATION OF THE PERTURBING MOMENTS DUE TO SOLAR RADIATION PRESSURE FORCES ACTING ON A BODY OF REVOLUTION

Ye. N. Polyakhova

(NASA-TT-F-15072) DETERMINATION OF THE PERTURBING MOMENTS DUE TO SOLAR RADIATION PRESSURE FORCES ACTING ON A BODY OF (Linguistic Systems, Inc., Cambridge, Mass.) 17 p HC $3.00 CSCL 03B G3/29
N73-33780
Unclas 19412

Translation of: "Opredeleniye vozmushchayushchikh momentov sil davleniya solnechnoy radiatsii, deystvuyushchikh na telo brashcheniya," Uchenyye Zapiski Leningradskogo Gosudarstvennogo Universiteta (Scientific Papers of the Leningrad State University) No. 363, Seriya Matematicheskikh Nauk (Mathematical Sciences Series) Issue 48, Trudy Astronomicheskoy Observatorii (Transactions of the Astronomical Observatory) Vol. 29, Leningrad, "Leningradskogo Universiteta" Press, 1973, pp. 152-163.

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 OCTOBER 1973

STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TT F-15,072		
4. Title and Subtitle: DETERMINATION OF THE PERTURBING MOMENTS DUE TO SOLAR RADIATION PRESSURE FORCES ACTING ON A BODY OF REVOLUTION		5. Report Date: OCTOBER 1973
		6. Performing Organization Code
7. Author(s): Ye. N. Polyakhova		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address: LINGUISTIC SYSTEMS, INC. 116 AUSTIN STREET CAMBRIDGE, MASSACHUSETTS 02139		11. Contract or Grant No.: ~~NASW-2482~~ NASW-2482
		13. Type of Report & Period Covered: TRANSLATION
12. Sponsoring Agency Name and Address: NATIONAL AERONAUTICS AND SPACE ADMINISTRATION WASHINGTON, D.C. 20546		
		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of: "Opredeleniye vozmushchayushchikh momentov sil davleniya solnechnoy radiatsii, deystvuyushchikh na telo brashcheniya," Uchenyye Zapiski Leningradskogo Gosudarstvennogo Universiteta (Scientific Papers of the Leningrad State University) No. 363, Seriya Matematicheskikh Nauk (Mathematical Sciences Series) Issue 48, Trudy Astronomicheskoy Observatorii (Transactions of the Astronomical Observatory) Vol. 29, Leningrad, "Leningradskogo Universiteta" Press, 1973, pp. 152-163.

16. Abstract

The simple approximate formulas are used for the determination of the perturbing torques due to solar radiation pressure forces for the summetrical satellite (body of revolution). Two types of such formulas are discussed for the forces which are directed: 1) to the light flow, 2) to normal the body surface. The general supposition is that the light flow lies in the plane of symmetry of satellite contained the spin axis. The formulas are used for the torques determination in the case of conical and paraboloidal satellites.

17. Key Words (Selected by Author(s))	18. Distribution Statement
	UNCLASSIFIED - UNLIMITED

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
UNCLASSIFIED	UNCLASSIFIED	15	

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Ye. N. Polyakhova

It is known that solar radiation pressure forces cause /153*
the appearance of perturbing moments relative to the center of mass of an artificial earth satellite (AES), the magnitude of which depends on the reflective capacity of its surface, on its shape, and on its orientation relative to the Sun. This moment is a consequence of the fact that the center of radiation pressure, as a rule, does not coincide with the center of mass of the AES. We will examine the problem of deriving simple approximate formulae, suitable for the evaluation of the perturbing moments due to the pressure forces on an AES having the form of a body of revolution.

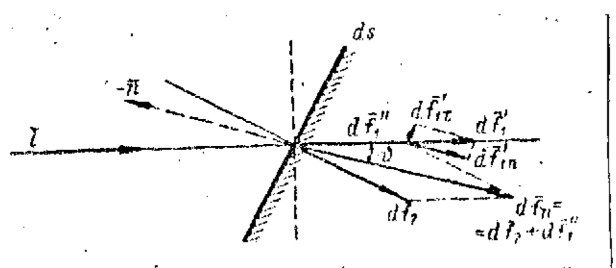

Figure 1. The geometry of incident and reflected rays.

We will examine a unit area, ds, oriented at the angle to the radiation flux $\bar{l}$ (Fig. 1) (the angle θ is the angle between the light ray $\bar{l}$ and the normal $\bar{n}$, directed toward the propagating light). Falling on it is a flux of radiating energy, S cos θ (S is a "solar constant" in the earth's orbit, a unit energy flux falling on an area normal to it). The corresponding The corresponding amount of motion, imparted to the unit

*Numbers in right-hand margin indicate pagination in foreign text.

area by the flux of solar radiation, is $Sc^{-1}\cos\theta$, where c is the speed of light. The modulus of the component of force $d\bar{f}_1$, acting in the direction of the flux, $\bar{\ell}$, is written thus:

$$df_1 = P_r \cos\vartheta\, ds \quad (P_r = Sc^{-1} = 0.46 \cdot 10^{-4}\,\mathrm{dyn.}\,cm^{-2}). \tag{1}$$

Besides the component $d\bar{f}_1$, a second component $d\bar{f}_2$ acts on the area, appearing as a result of the action of the reflected ray, pointing in the direction opposite it; and having a modulus

$$df_2 = \varepsilon P_r \cos\vartheta\, ds, \tag{2}$$

where ε, the coefficient of reflection, is the ratio of the densities of reflected energy and incident flux. In turn, one may represent the force $d\bar{f}_1$ as the sum of two components, co-linear with the vector $\bar{\ell}$:

$$d\bar{f}_1 = d\bar{f}_1' + d\bar{f}_1'', \quad df_1 = df_1' + df_1'' \tag{3}$$

where

$$df_1' = (1-\varepsilon) P_r \cos\vartheta\, ds, \quad df_1'' = \varepsilon P_r \cos\vartheta\, ds, \tag{4}$$

thus separating it from the component df_1'', equal to df_2. Considering the symmetry of the incident and reflected rays relative to the normal $\bar{n}$, one may conclude that the sum of the vectors, $d\bar{f}_1''$ and $d\bar{f}_2$, which are of equal modulus, is positioned, in fact, along the normal $\bar{n}$, and its modulus will be /154

$$df_n = 2\varepsilon P_r \cos^2\vartheta\, ds. \tag{5}$$

Thus, we may write the total force $d\bar{f}$ as the sum of the component from the impact $d\overline{f}_1'$ (partial) and the normal component $d\overline{f}_n$

$$d\overline{f} = d\overline{f}_1' + d\overline{f}_n. \tag{6}$$

Another representation of $d\overline{f}$ is possible: if the remaining force $d\overline{f}_1'$, directed along $\overline{l}$ is broken into tangent and normal components, $d\overline{f}_{1\tau}'$ and $d\overline{f}_{1n}'$ respectively (Fig. 1), then one can represent the force df as the sum of the summed normals and the tangential component:

$$d\overline{f} = (d\overline{f}_{1n}' + d\overline{f}_n) + d\overline{f}_{1\tau}'.$$

We will now consider the AES (a body of revolution) and assign to it the following system of coordinates, Oxyz: O is the origin on the axis of revolution; the z axis runs along the axis of revolution and is directed so that the inner normal to the surface forms an acute angle with the z axis; the yz plane is oriented so that it always contains the light flux $\overline{l}$. Projecting (6) onto the coordinate axes, we get

$$df_x = (1-\varepsilon) P_r \cos\theta \cos(\overline{l}, \widehat{\overline{x}})\, ds + 2\varepsilon P_r \cos^2\theta \cos(\overline{n}, \widehat{\overline{x}})\, ds, \tag{7}$$

and so forth.

Setting up the integral formulae for the moments of the radiation pressure forces, we also write them as the sum

$$M_x = \iint\limits_s \left(y \frac{df_z}{ds} - z \frac{df_y}{ds} \right) ds = M_x' + M_x'',$$

and so forth (8)

where
$$M_x' = (1-\varepsilon) P_r \iint\limits_s [y \cos\theta \cos(\overline{l}, \widehat{\overline{z}}) - z \cos\theta \cos(\overline{l}, \widehat{\overline{y}})]\, ds,$$

$$M_x'' = 2\varepsilon P_r \iint [y \cos^2\theta \cos(\overline{n}, \widehat{\overline{z}}) - z \cos^2\theta \cos(\overline{n}, \widehat{\overline{y}})]\, ds. \tag{8'}$$

Formulae (8) and (8') make it possible to define the moments of the pressure forces if cos θ and the six direction cosines

of the incident ray $\bar{\ell}$ and of the normal to the surface $\bar{n}$, cos $(\widehat{\bar{\ell},x})$ . . . cos $(\widehat{\bar{n},z})$, are known. The direction cosines of $\bar{n}$ depend only on the shape of the surface and may be calculated /155
on the basis of the equation of the surface, (x,y,z) = 0, according to the formula

$$\cos(\widehat{\bar{n}, x}) = \frac{\varphi_x}{\sqrt{\varphi_x'^2 + \varphi_y'^2 + \varphi_z'^2}}. \tag{9}$$

Considering that products of the type cos $(\widehat{\bar{n},\bar{z}})$ds represent the area of the projection of area ds onto the xy plane, i.e., ds_{xy} = dxdy, we will write an expression for the moments in the following manner:

$$\begin{gathered} M_x'' = 2\varepsilon P_r(I_{1x} - I_{2x}), \\ I_{1x} = \iint\limits_{s_{xy}} y \cos^2\vartheta \, dxdy, \ I_{2x} = \iint\limits_{s_{xz}} z \cos^2\vartheta \, dzdx. \end{gathered} \tag{10}$$

If cos Θ = const. (a plane), then it follows that one can calculate these integrals in quadratics, substituting the corresponding coordinate function for s_{xy}, s_{xz} and s_{yz}. In the general case of an AES of arbitrary shape, it follows to evaluate beforehand an approximate mean value of $\cos^2\Theta$.

For the moments M_x' from formula (8) one may write analogous formulae

$$\begin{gathered} M_x' = (1 - \varepsilon) P_r (K_{1x} - K_{2x}), \\ K_{1x} = \iint\limits_{s} y \cos\vartheta \cos(\widehat{\bar{l}\, \bar{z}})\, ds, \ K_{2x} = \iint\limits_{s} z \cos\vartheta \cos(\widehat{\bar{l}, \bar{y}})\, ds. \end{gathered} \tag{11}$$

For the representation in (11) it is more convenient to use the expression

$$\cos\vartheta = \cos(\widehat{\bar{l}\, \bar{x}}) \cos(\widehat{\bar{n}\, \bar{x}}) + \cos(\widehat{\bar{l}\, \bar{y}}) \cos(\widehat{\bar{n}\, \bar{y}}) + \cos(\widehat{\bar{l}\, \bar{z}}) \cos(\widehat{\bar{n}\, \bar{z}}). \tag{12}$$

Breaking each integral of type K_{1x} into three integrals, we get

$$K_{1x} = \cos(\widehat{\bar{l}\,\bar{z}}) \sum_{n=1}^{3} K_{1x}^{n}, \quad K_{2x} = \cos(\widehat{\bar{l}\,\bar{y}}) \sum_{n=1}^{3} K_{2x}^{n}, \tag{13}$$

where

$$\begin{aligned} K_{1x}^{1} &= \cos(\widehat{\bar{l},\,\bar{x}}) \iint y\,dy\,dz; \quad K_{2x}^{1} = \cos(\widehat{\bar{l}\,\bar{x}}) \iint z\,dy\,dz, \\ K_{1x}^{2} &= \cos(\widehat{\bar{l}\,\bar{y}}) \iint y\,dz\,dx; \quad K_{2x}^{2} = \cos(\widehat{\bar{l}\,\bar{y}}) \iint z\,dz\,dx, \\ K_{1x}^{3} &= \cos(\widehat{\bar{l}\,\bar{z}}) \iint y\,dx\,dy; \quad K_{2x}^{3} = \cos(\widehat{\bar{l},\,\bar{z}}) \iint z\,dx\,dy. \end{aligned} \tag{13'}$$

It follows that the direction cosines $\cos(\widehat{\bar{\ell}\,\bar{x}})$ may be calculated earlier according to the conditions of illumination, and the integrals may be calculated in quadratics or according to the formula of static moments:

$$\iint_{s_{xy}} y\,dx\,dy = y_c s_{xy},$$

where y_c is the ordinate of the center of mass of the area s_{xy} (the projection of the illuminated part of the surface of the AES onto the coordinate plane). /156

2. DETERMINATION OF THE MOMENTS, ACTING ON A CONICAL AND ON A PARABOLIC AES

We will look at a conical AES, at the top O of which we will place the origin of the coordinates. Let the dimension R and $H = Rp^{-1}$ be given so that the angle of the semi-span of the cone ϕ may be taken as known. Let the ray $\bar{\ell}$ lie in the yz plane, so that

$$\cos(\widehat{\bar{l}\,\bar{x}}) = 0, \quad \cos(\widehat{\bar{l}\,\bar{y}}) = -\cos\beta, \quad \cos(\widehat{\bar{l}\,\bar{z}}) = -\sin\beta. \tag{14}$$

The problem consists of determining the moments of the radiation pressure forces for the conical surface $pz - \sqrt{x^2 + y^2} = z \tan\phi - r^2 = 0$, the direction cosines of the normals to the surface being:

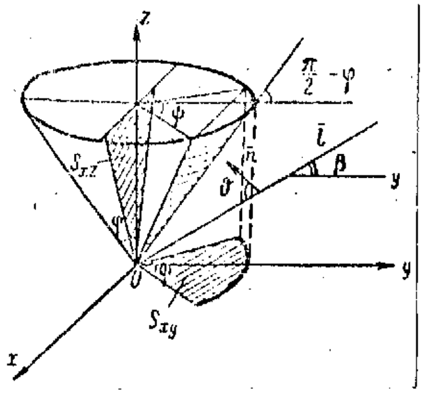

Figure 2. A cone of revolution.

$$\cos(\widehat{\bar{n}\,\bar{x}}) = -\frac{x}{r\sqrt{p^2+1}},$$
$$\cos(\widehat{\bar{n}\,\bar{y}}) = -\frac{y}{r\sqrt{p^2+1}}, \qquad (15)$$
$$\cos(\widehat{\bar{n}\,\bar{z}}) = \frac{p}{\sqrt{p^2+1}},$$

while for the yz plane containing the ray $\bar{l}$ we immediately get $\cos(\bar{n}\cdot\bar{x}) = 0$. It is also easy to determine the areas of the projection of the illuminated part of the surface onto the coordinate planes.

$$s_{xy} = R^2\psi, \quad s_{xz} = RH\sin\psi.$$

We set up the expression for M'_x:

$$M'_x = (1-\varepsilon)P_r\left[\cos^2(\widehat{\bar{l}\,\bar{z}})\iint y\,dxdy - \cos^2(\widehat{\bar{l}\,\bar{y}})\iint z\,dzdx\right] = (1-\varepsilon)P_r\left[\sin^2\beta\, I_1 + \cos^2\beta\cdot I_2\right], \qquad (16)$$

where the integrals I_1 and I_2 must be calculated in the sector s_{xy} and in the triangle s_{xz}, as a result of which we get

$$I_1 = \int_0^R \rho^2 d\rho \int_{-\psi}^{+\psi}\cos\gamma\, d\gamma = \frac{2}{3}R^3\sin\psi,$$

where $\psi = \operatorname{arctg}\sqrt{\operatorname{ctg}^2\varphi\cdot\operatorname{ctg}^2\beta - 1}$,

$$I_2 = \int_0^H z\,dz \int_{-l(z)}^{+l(z)} dx = 2\int_0^H z\,l(z)\,dz = \frac{2}{3}RH^2\sin\psi,$$

where $l(z) = \frac{Rz\sin\psi}{H}$ is half of the line passing through the triangle s_{xz} parallel to its base (Fig. 2). From here /157

$$M'_x = (1-\varepsilon)P_r\frac{2}{3}\sin\psi\,(R^3\sin^2\beta + RH^2\cos^2\beta). \qquad (17)$$

By analogous means we may get the formulae for M''_x:

$$M_x'' = 2\varepsilon P_r (Q_1 + Q_2),$$
$$Q_1 = \iint \cos^2\theta \, y \, dx dy, \quad Q_2 = \iint \cos^2\theta \, z \, dx dz. \tag{18}$$

We set up the expression for cos θ according to formula (12)

$$\cos\theta = \frac{1}{\sqrt{p^2+1}} \left(\frac{y}{r} \cos\beta - p \sin\beta \right), \tag{19}$$

from which it is evident that cos θ>0 for $\frac{x}{\sqrt{x^2 + y^2}}$ greater than p tan β, as a result of which the equation of the terminator

$$\frac{x}{y} = \frac{\sqrt{1 - p^2 \operatorname{tg}^2\beta}}{p \operatorname{tg}\beta} = \sqrt{\operatorname{ctg}^2\varphi \cdot \operatorname{ctg}^2\beta - 1} = \operatorname{tg}\psi$$

represents an equation of two generatrices of the cone, the projections onto the xy plane tilted from the y axis to angles of ψ and −ψ. Squaring (19), we derive two expressions for the integration with respect to dxdy and dzdz:

$$\cos^2\theta = A^2 \frac{y^2}{r^2} - 2AB \frac{y}{r} + B^2 = -\frac{A^2}{p^2} \cdot \frac{x^2}{z^2} - 2\frac{AB}{p} \cdot \frac{\sqrt{p^2 z^2 - x^2}}{z} + (A^2 + B^2);$$

where

$$A = \frac{\cos\beta}{\sqrt{p^2+1}}, \quad B = \frac{p \sin\beta}{\sqrt{p^2+1}}.$$

Carrying out the integration, we receive for Q_1 and Q_2

$$Q_1 = \frac{1}{3} A^2 R^3 \left(2\sin\psi - \frac{2}{3}\sin^3\psi\right) - \frac{2}{3} ABR^3 \left(\psi + \frac{1}{2}\sin 2\psi\right) + \frac{2}{3} B^2 R^3 \sin\psi, \quad Q_2 = -\frac{2}{9} A^2 R^3 p^{-2} \sin^3\psi - \frac{2}{3} ABH^3 p \left(\psi + \frac{1}{2}\sin 2\psi\right) + \frac{2}{3} (A^2 + B^2) RH^2 \sin\psi.$$

Thus the problem of determining M_x'' according to formula (18) is solved.

If several averaged values of $\cos^2\theta$ for the illumination of the surface are known beforehand, then, signifying it by σ, one may write

$$M_x'' = 2\varepsilon P_r \sigma \cdot \frac{2}{3} \sin\psi \,(R^3 + RH^2).$$

For σ it is convenient to take a constant value of /158
$\cos^2\theta = (A-B)^2$ for the generatrix of the cone, lying in the plane of the light flux $\overline{\ell}$.

If it is necessary to move the moment to the center of mass of the cone, that follows by the use of the formula

$$\overline{M}_C = \overline{M}_0 - (\overline{OC} \times \overline{R}) = \overline{M}_0 - \begin{vmatrix} \overline{i} & \overline{j} & \overline{k} \\ 0 & 0 & (OC)_z \\ 0 & R_y & R_z \end{vmatrix},$$

where $\overline{R} = \sum_i \overline{f}_i$ is the main vector of the forces of pressure, thus for a body of revolution, it is sufficient to determine only its component $R_y = R_y' + R_y''$ according to the formulae

$$R_y' = (1-\varepsilon) P_r \iint \cos\theta \cos(\widehat{\overline{l}\,\overline{y}})\, ds, \quad R_y'' = 2\varepsilon P_r \iint \cos^2\theta \cos(\widehat{\overline{n}\,\overline{y}})\, ds,$$

which for a cone takes on the following form:

$$\begin{aligned} R_y' &= -(1-\varepsilon) P_r \,[R^2\psi \sin\beta\cos\beta + RH \sin\psi \cos^2\beta], \\ R_y'' &= 2\varepsilon P_r \Big[\frac{A^2R^3}{3p^2H} \sin^3\psi + ABH^2 p \Big(\psi + \frac{1}{2}\sin\psi\Big) - \\ &\quad - (A^2+B^2) RH \sin\psi \Big], \end{aligned} \qquad (20)$$

or for an approximate variation

$$R_y'' = -2\varepsilon P_r \sigma R H \sin\psi.$$

The magnitude of $(OC)_z$ will vary depending on the construction of the cone: for a continuous, uniform cone, $(OC)_z = OC = \frac{3}{4} H$; for a hollow and open cone, $OC = \frac{2}{3} H$; for a hollow and closed cone, $OC = \frac{H}{1+R} (\frac{2}{3}\ell + R)$, where ℓ is the length of the generatrix.

For a closed cone, it is easy to determine the moments of forces acting on the base:

$$\begin{aligned} M'_x &= (1-\varepsilon) P_n \sin\beta\cos\beta\,\pi R^2 H,\quad M''_x = 0,\\ R'_y &= -(1-\varepsilon) P_r \sin\beta\cos\beta\,\pi R^2,\quad R''_y = 0. \end{aligned} \tag{21}$$

We will now look at a parabolic AES, oriented in a manner similar to the cone (Fig. 3) with the equation of the surface $2pz - (x^2 + y^2) = 0$, where $p = \frac{R^2}{2H}$ is the parameter of the generatrix of the parabola. The direction cosines of the normal to the paraboloid are such

$$\cos(\widehat{\bar{n}\,\bar{x}}) = \frac{-x}{\sqrt{x^2+y^2+z^2}},\quad \cos(\widehat{\bar{n}\,\bar{y}}) = \frac{-y}{\sqrt{x^2+y^2+z^2}},$$
$$\cos(\widehat{\bar{n}\,\bar{z}}) = \frac{p}{\sqrt{x^2+y^2+z^2}},$$

thus for the yz plane, containing the light flux $\bar{l}$, we have /159
$\cos(\widehat{\bar{n}\,\bar{x}}) = 0$. Correspondingly for $\cos\theta$ we get

$$\cos\theta = \frac{1}{\sqrt{x^2+y^2+z^2}}(y\cos\beta - p\sin\beta). \tag{22}$$

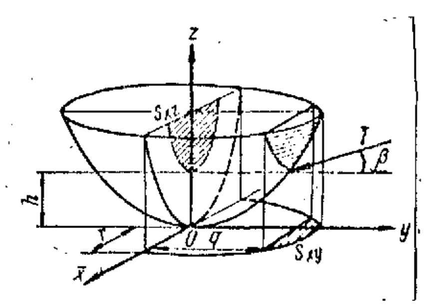

Figure 3. Paraboloid of revolution.

The equation of the terminator in the given case is the equation of the parabola $x^2+p^2\tan^2\beta = 2pz$, the top of which lies above the xy plane at a height $h = \frac{1}{2}p\tan^2\beta$. The abscissas of the points of the illuminated surface and of angle β satisfy the conditions

$$-\sqrt{2p(H-h)} \leq x \leq \sqrt{2p(H-h)},\quad -\frac{\pi}{2} \leq \beta \leq \operatorname{arctg}\frac{y}{p}.$$

For β greater than $\arctan\frac{y}{p}$ not one point on the outer surface is illuminated. Deriving the integral with respect to dxdy and dxdz for the parabolic surface, we get

$$I_1 = \frac{2}{3}R^3\left[1-\left(\frac{q}{R}\right)^2\right]^{3/2}, \text{ где } q = p\,\mathrm{tg}\,\beta,$$
$$I_2 = 4\sqrt{2p}\left[\frac{1}{5}(H-h)^{5/2} + \frac{1}{3}h(H-h)^{3/2}\right], \qquad (23)$$

so that formula (16) is again used for the calculation of M_x'.

Moving on to the calculation of M_x'', we note that the precise calculation of the integrals Q_1 and Q_2 from formula (18) is done in this case with too much difficulty, and for an approximate evaluation it is sufficient to use an averaged value of $\cos^2\theta$, which for a paraboloid will not be constant for the directrix as in the case of the cone. For the determination of σ we square (22) and setting x=0 (the directrix, lying in the plane of flux), we get the expression

$$\cos^2\theta = \frac{1}{y^2+p^2}\left(y^2\cos^2\beta - 2yp\sin\beta\cos\beta + p^2\sin^2\beta\right),$$

averaging which with respect to the directrix, we get the formula for σ:

$$\sigma = \frac{1}{R}\int_0^R \cos^2\theta\,dy = \cos^2\beta + pR^{-1}(\sin^2\beta -$$
$$-\cos^2\beta)\,\mathrm{arctg}\,(pR^{-1}) - pR^{-1}\sin\beta\cos\beta\ln\frac{R^2+p^2}{p^2}.$$

Thus, the problem concerning the determination of M_x'' may be considered solved. /160

For the area of the projection it is easy to get

$$s_{xy} = \frac{1}{2}\pi R^2 - q\sqrt{R^2-q^2} - R^2\arcsin\frac{q}{R},$$
$$s_{xz} = \frac{4}{3}\sqrt{2p}\,(H-h)^{3/2},$$

so that for transfering the moment to the center of mass, it follows to use the formulae

$$R_y' = -(1-\varepsilon)P_r(\sin\beta\cos\beta\cdot s_{xy} + \cos^2\beta\cdot s_{xz}), \quad R_y'' = -2\varepsilon P_r\sigma s_{xz},$$

$(OC_2)=OC=\frac{3}{5}H$ (a continuous uniform paraboloid).

The following model problem will be carried out as an example of the calculation of the moments of the radiation pressure forces examined: determining the moments for two continuous uniform bodies, a cone and a paraboloid with identical linear dimensions: $R = 1$ m, $H = 10$ m while $\varepsilon = 0.5$ over the entire range $0 \leq \beta \leq 360^{\circ}$. The results of the calculation of the magnitude of M_x' are presented in Fig. 4. Shown in Fig. 5 are the moments M_x'' for the variation of approximate evaluations of $\cos^2\theta$ along the directrix of the body. Shown in Fig. 6 is the total moment $M_x(\sigma)$, calculated relative to the top of the body of revolution, and the moment, calculated relative to the base. Presented in Fig. 7 are the moments transferred to the center of mass (the continuous line corresponds to the cone; the dashed line, to the paraboloid).

CONCLUSION

We have examined the problem concerning the analytical determination of the moments of the solar ray pressure forces on a body of revolution, located in the radiation flux. General integration formulae were worked out separately for the moments of the pressure forces directed along the ray of light, and of the forces directed along the normal to the illuminated surfaces. The acquired formulae, for the most part, are convenient for approximate evaluations and rough estimates of the boundaries of change of the magnitude of the perturbing moment of an AES for the whole range of angles of illumination during its movement along its orbit. The acquired results, presented graphically, allow one to make the basic conclusion that an elongated body, located in a flux of radiation, will try to align itself lengthwise along the flux as a result of the action of the perturbing moment. Here, the perturbing moment of the pressure forces causes the turning of the body about the Ox axis, perpendicular to the radiation flux $\bar{\ell}$ and simultaneously perpendicular to the axis of revolution Oz.

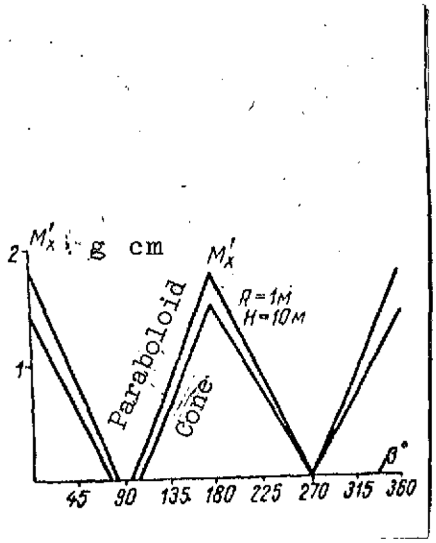

Figure 4. The moments M_x' as a function of angle of illumination β.

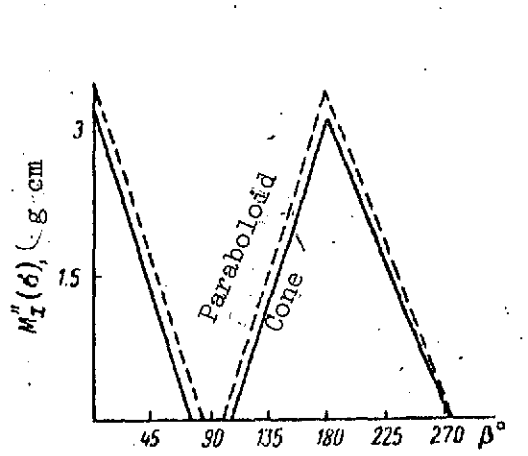

Figure 5. The moments M_x' as functions of angle of illumination β.

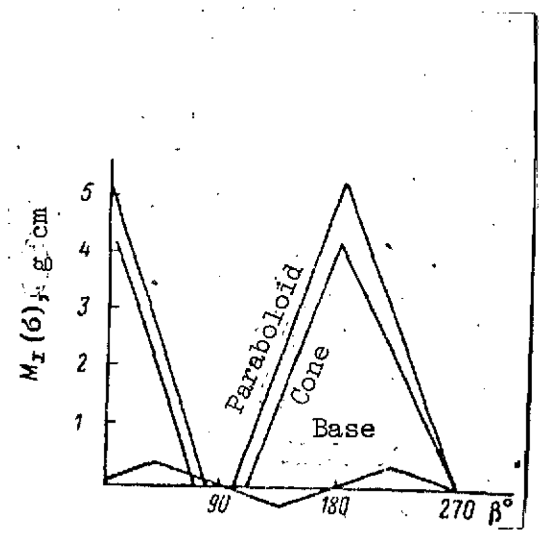

Figure 6. Total moments M_x as functions of angle of illumination β.

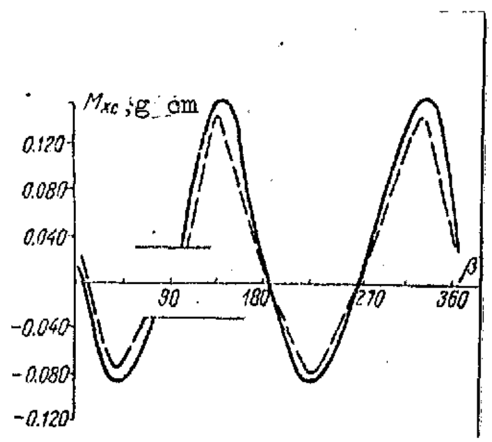

Figure 7. The moments M_{xc} (moved to the center of mass) as functions of angle of illumination β.

In the stated formulation of the problem, we confine ourselves to the examination of the dependence of the moment of the pressure forces on the angle of illumination, β, by which /162
we mean the angle between the ray $\bar{\ell}$ and the coordinate axis which is perpendicular to the axis or revolution of the body. For each specific body, formulae were worked out for the connection of angle β with the angle of incidence θ , formed by the ray $\bar{\ell}$ and the inner normal $\bar{n}$, however, we did not connect the angle β with time, i.e. with a specific position of the AES in orbit and its orientation relative to the Sun. For the solution of the problem of determining the moments at a given moment of time, it is necessary to have available the dependence $\beta = \beta(t)$, for which it follows that one must determine the direction cosines of $\bar{\ell}$ with the axes of the orbital system of coordinates according to the mutual positions or the AES of the Sun and one must determine the orientation of the axis of revolution of the AES relative to this same system along the given Eulerian angles. If, for instance, the AES is gravitationally oriented, then the main axis of inertia (in our case, of revolution) is directed along a geocentric radius vector.

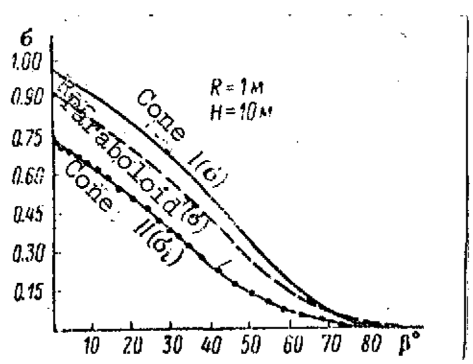

Figure 8. Values of the parameter σ as a function of the angle of illumination β.

The presented formulae for the moments were set up approximately under two basic simplifying assumptions. The first of these, as was already pointed out, is that the light flux $\bar{\ell}$ is propogating in the plane of symmetry of the arbitrarily oriented AES, containing the axis of revolution: as a consequence of the symmetry of forces, determined by the stated assumption, the general integral formulae are substantially simplified. The second simplifying assumption is used only during the determination of the moments M_x'' and relates to the magnitude of $\cos^2\theta$, which for sufficiently simple bodies (a sphere, cylinder, or cone) may be calculated exactly by means of the calculation of the corresponding surface integrals. However, for more complex bodies, for instance, for a paraboloid, the calculation of these integrals is much too time consuming and it is more convenient to use a value of σ averaged along the directrix, placing it outside the integral sign. The error, appearing as a result of such a substitute, may be evaluated from the graphs in Fig. 8, where the following curves are presented for selected model problems: "cone I(σ)" is the curve of σ based on the approximating relation $\sigma = \sigma(\beta) = \cos^2\theta = (A-B)^2$ (a directrix, lying in the plane of the flux); "cone II$(i)_\sigma$ is the curve of $\sigma = \sigma_i$, obtained according to the formula

$$\sigma = \sigma_i = \frac{Q_1}{I_1} = \frac{Q_2}{I_2},$$

in which Q_1 ... I_2 are calculated exactly with the help of an integration (the sumbol σ_i corresponds to the integrated mean value of σ). A comparison of the curves shows that the use of an approximate σ gives an increased value of the moment to a maximum of 1.3 times in comparison with the exact moment. The curve "paraboloid (σ)" corresponds to the approximate variation of σ (the exact value we will not derive), however, keeping in mind the geometrical properties of a parabolic surface, one may assert that the expected error in the determination of the moment will not exceed the error for the cone. /163

All the ennumerated assumptions lead to a substantial simplification of the formulae, as a result of which all the integrals are expressed in quadratics while the rigorous analytic determination of the moments of the radiation pressure forces is extremely time-consuming even for simple surfaces and does not have any advantage over numerical methods.